HYBRID COMBUSTION THEORY
(DIFFUSION LIMITED MODEL)

Development of the hybrid burning law:

Energy balance at the fuel surface: (Steady-state)

$$\dot{Q}_w = \dot{m}_f h_v = \rho_f \dot{r}\, h_v = \left(\rho v\right)_w h_v$$

$\dot{Q}_w$ = Total heat flux to the wall

h_v = Effective heat of gasification (Heating of the solid fuel grain + Heat of evaporation and melting + Heat of reaction for degradation of the polymer)

Conductive Heat Transfer:

- Assume that $\dot{Q}_w = \dot{Q}_c = -\left(\dfrac{k}{c_p}\dfrac{\partial h}{\partial y}\right)_w$

Define a Stanton number

$$C_H \equiv \frac{\dot{Q}_c}{\rho_b u_b \Delta h} \qquad \text{where} \qquad \Delta h = h_b - h_w$$

Combine

$$\rho_f \dot{r} = \frac{\dot{Q}_c}{h_v} = C_H \rho_b u_b \frac{\Delta h}{h_v}$$

- What is C_H?

$$C_H = f(C_f)$$

(Note that there is extensive information on C_f in the TBL literature.)

- Reynods analogy between the flame and the wall. (No chemical rxns beneath the flame)

$$\frac{\dot{Q}_c}{\Delta h} = \frac{\tau_w}{u_b}$$

- Note that

$$\tau_w = 0.5 C_f \rho_e u_e^2$$

Combine to obtain

$$C_H = 0.5 C_f \frac{\rho_e u_e^2}{\rho_b u_b^2} \qquad \text{and} \qquad \dot{r} = \frac{C_f \rho_e u_e B}{2\rho_f}$$

where the "Blowing parameter" is defined as

$$B \equiv \frac{2(\rho v)_w}{\rho_e u_e C_f} = \frac{u_e}{u_b}\frac{\Delta h}{h_v}$$

- The skin friction coefficient for flat plate TBL and without blowing

$$C_{f0} = 0.06\,\mathrm{Re}_x^{-0.2} \qquad \text{where} \qquad \mathrm{Re}_x = \frac{\rho_e u_e x}{\mu_e}$$

Substitute to get

$$\dot{r} = 0.03\frac{\rho_e}{\rho_f}u_e\,\mathrm{Re}_x^{-0.2}\left(\frac{C_f}{C_{fo}}\right)B$$

- Correction for blowing $\left(C_f / C_{f0}\right)$. (Blocking effect.)

  - Lee's "film theory" predicts

$$C_f \Big/ C_{f0} = \ln\left[\frac{(1+B)}{B}\right] \qquad \text{(B<5)}$$

  - Marxman derived the formula

$$C_f / C_{f0} = 1.2B^{-0.77} \qquad \text{(5<B<100)}$$

- Introduce the mass flux $G \equiv \rho_e u_e$ and rearrange to obtain the hybrid regression rate expression

$$\dot{r}(x) = \frac{0.036}{\rho_f \mu_e^{-0.2}} G^{0.8} x^{-0.2} B^{0.23}$$

Calculation of B:

- Combustion model is required.

- General solution is obtained by solving the gas-phase field equations with regression rate equation as one of the boundary conditions. (Difficult problem)

- Marxman obtained an approximate solution using the mixing length concept.

- For $L/D < 25$ B does not change significantly with x.

(L: Length of the grain, D: Hydraulic diameter) Thus threat B as a constant for a given oxidizer/fuel selection.

- B is a dual parameter: 1) Thermochemical property of the selected propellant 2) Aerodynamic property (Similarity parameter of the TBL profile)

- In the nondimensional form the result is

$$\dot{r}_{nd} = \frac{2 \rho_f \dot{r}}{G C_{fo}} = 1.2 B^{0.23} = cons.$$

- Regression rate is not a strong function of B. Mass flux has the most significant effect on the burning rate.

$$\dot{r}(x) = A G^{0.8} x^{-0.2}$$

where A is

$$A = \frac{0.036}{\rho_f \mu_e^{-0.2}} B^{0.23}$$

- A can be assumed to be constant as a first order approximation
- For purely convective systems, regression rate is not a function of pressure.

Radiative Heat Transfer:

- Simple model: Grain as gray body, flame zone as the radiative continuum.

The radiative heat transfer can be written as

$$\dot{Q}_r = \sigma \varepsilon_w \left(\varepsilon_g T_r^4 - T_w^4 \right) \text{ and } \varepsilon_g = 1 - e^{-\alpha N z} = F(P_c)$$

- Here ε_w and T_r (Effective radiation temperature) depend on the propellant combination.

Combined Heat Transfer:

- Superposition is not possible since $\dot{Q}_r$ and $\dot{Q}_c$ are coupled through the blocking effect.

- The following formula can be derived

$$\dot{r} = \frac{\dot{Q}_c}{\rho_f h_v}\left[e^{-\dot{Q}_r/\dot{Q}_c} + \frac{\dot{Q}_r}{\dot{Q}_c} \right]$$

- When $\dot{Q}_r << \dot{Q}_c$

$$\dot{r}(x) = A\,G^{0.8} x^{-0.2} + \frac{\dot{Q}_r}{\rho_f h_v}$$

- For most fuels without metal additives, the radiative contribution can be ignored.

Hybrid Burning Rate Law:

- If we average the regression rate expression over the grain length and firing period, we obtain the space-time averaged regression rate expression, namely the burning rate law for hybrids.

$$\bar{\dot{r}} = a\,G_o^n \qquad \text{n} \sim 0.5\text{-}0.8$$

G_o: Oxidizer mass flux ($\dot{m}_o / A_{po}$)

$\dot{m}_o$: Oxidizer mass flow rate

A_{po}: Average fuel port area

a, n: Empirical constants

- Note that the burning rate law for solid rockets has the form

$$\bar{\dot{r}} = c\,P_c^n$$

Typical Values for Combustion Parameters:

$B \approx 7-15$ $\qquad$ $O/F \big/ (O/F)_{stoic} < 1$

$T_b \approx 1500\ \text{K} \quad T_w \approx 600-800\ \text{K}$

Flame height: 0.15- 0.2 BL thickness

Flame thickness: 0.1 BL thickness