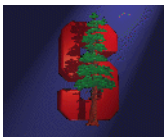

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Lecture 10
Hybrid Rocket Propulsion
Design Issues

Prepared by
Arif Karabeyoglu

Department of Aeronautics and Astronautics
Stanford University

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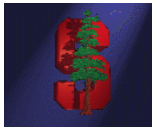


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Axial Flow Hybrid Rocket

- For an axial flow hybrid the oxidizer flows parallel to the axis of the propulsion system through a cylindrical cavity(s) inside fuel grain
- The cross sectional shape of the cylindrical opening (port) can be
 - Circle, Triangle, D-shape or any other complex form
- There could be multiple cylindrical openings (ports), multi-port hybrid
 - Two ports: Double-D
 - Four ports: Quad
 - Five ports: Quad+1
 - Larger number of ports utilizes single or double row wagon wheel configuration
 - AMROC motor :15+1 ports
- Fuel utilization (minimizing the sliver fraction) dictates the shapes of the ports once the number of ports and overall port configuration is selected
- The fuel sliver fraction increases with the increasing number of corners
- Note that the hydraulic diameters of the ports must be matched for even burning
- Axial variation of the port geometry is also possible
- Most simple port design is a single circular geometry. This is the most efficient shape for fuel utilization. No corners
- We will limit the discussions to single port hybrids



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Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

- The local instantaneous regression rate expression:

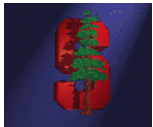
$$\dot{r}(x, t) = a G^n x^m$$

- Note that classical theory predicts $n=0.8$ and $m=-0.2$
- First consider the axial variation of regression rate for a given instant in time. For simplicity assume that the port shape and hydraulic diameter is independent of x .
- The axial mass balance in the fuel port yields (C_p is the circumference)

$$\dot{m}(x) = \dot{m}_{ox} + \int_0^x \rho_f C_p \dot{r}(x') dx'$$

- Convert to flux (A_p is the port area) and substitute the regression rate expression

$$G(x) = G_{ox} + \frac{a \rho_f C_p}{A_p} \int_0^x G^n x'^m dx'$$



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Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

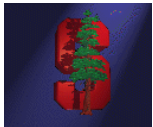
- Convert the integral equation to a differential equation

$$\frac{dG}{dx} = a \rho_f \frac{C_p}{A_p} G^n x^m$$

- Integrate by the separation of variables

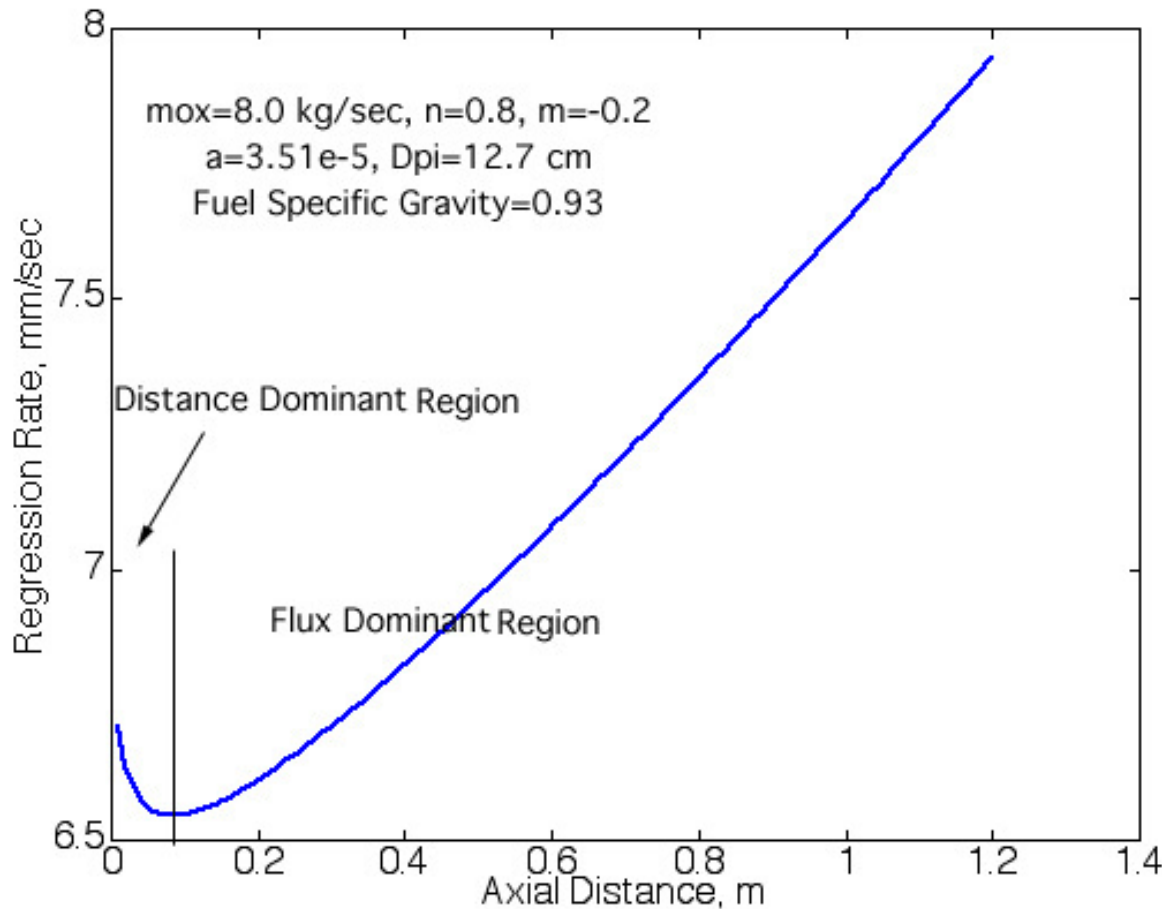
$$G(x) = \left(G_{ox} + a \rho_f \left(\frac{1-n}{1+m} \right) \frac{C_p}{A_p} x^{1+m} \right)^{1/1-n}$$

- Note that $G_{ox} = \frac{\dot{m}_{ox}}{A_p}$
- For circular port $\frac{C_p}{A_p} = \frac{4}{D_p}$
- To obtain regression rate substitute the flux expression into the regression rate equation

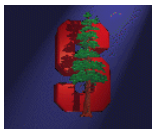


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Axial Flow Hybrid Rocket – Axial Variation of Regression Rate



- Constant port area assumption implies that the derived formula for the axial variation of the regression rate is only valid at $t=0$
- The regression rate variation can be more than 20%
- Axial change in port diameter more than 10% is rarely observed
- This due to the self correcting behavior: As one part of the port opens up more, the local flux decreases, resulting in a decrease in the regression rate
- Typically space averaged port diameter is used in the calculations



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Axial Flow Hybrid Rocket – Derivation of the Design Equations

- As discussed in the previous section, the axial variation of the port diameter and the regression rate will be ignored. Introduce the space averaged port diameter and the regression rate
- Use the simplified space averaged regression rate expression which is assumed to be valid at any instant of the hybrid operation

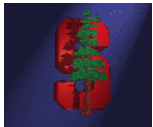
$$\dot{r}(t) = a G_{ox}^n$$

- Using the definition of the regression rate the dynamic equation for the space averaged port diameter can be written as

$$\frac{dD_p}{dt} = 2 \dot{r} = 2 a G_{ox}^n$$

- For a circular port hybrid

$$\frac{dD_p}{dt} = \frac{2^{2n+1} a \dot{m}_{ox}^n}{\pi^n D_p^{2n}}$$



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Axial Flow Hybrid Rocket – Derivation of the Design Equations

- For constant oxidizer flow rate, this expression can be integrated to obtain the port diameter as a function of time

$$D_p(t) = \left[D_{pi}^{2n+1} + \frac{(2n+1)2^{2n+1} a}{\pi^n} \dot{m}_{ox}^n t \right]^{1/(2n+1)}$$

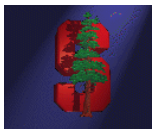
- The instantaneous flux, regression rate, fuel mass flow rate, O/F can be calculated from

$$G_{ox}(t) = \frac{4\dot{m}_{ox}}{\pi D_p^2} \quad \dot{r}(t) = a G_{ox}^n \quad \dot{m}_f(t) = \rho_f \pi D_p L \dot{r} \quad \frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_f}$$

- Based on O/F, total mass flow rate and nozzle geometry, one can determine the chamber pressure and thrust as a function of time

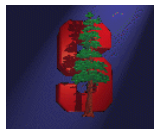
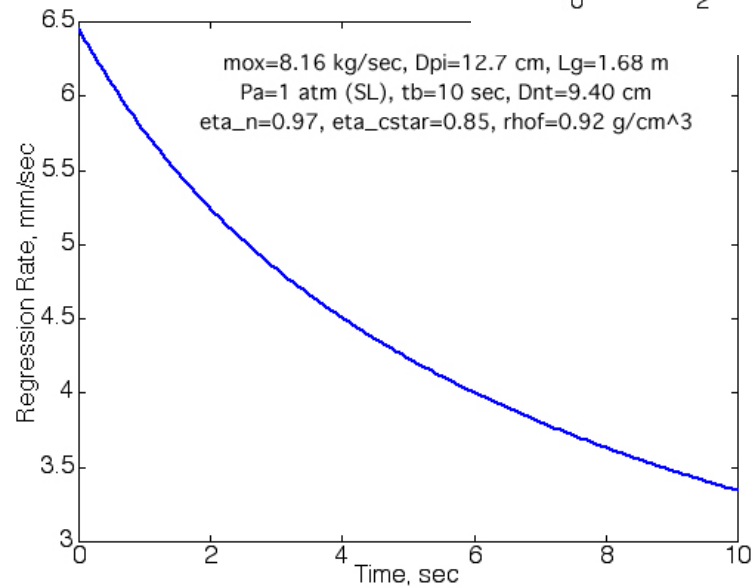
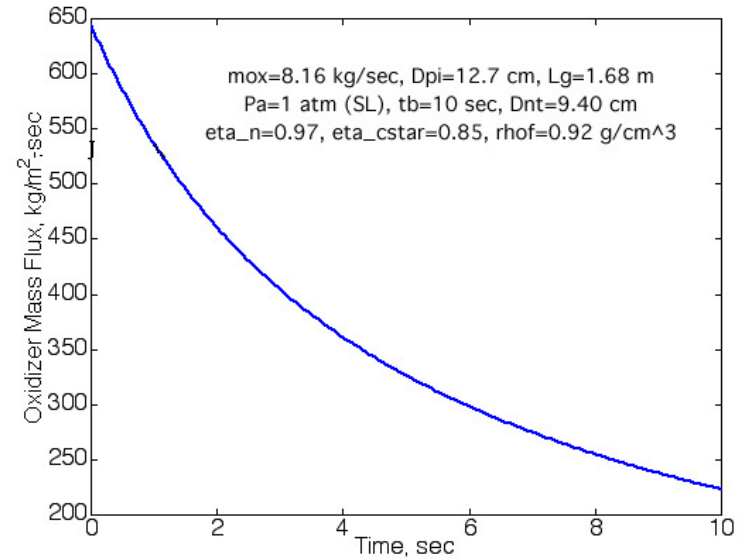
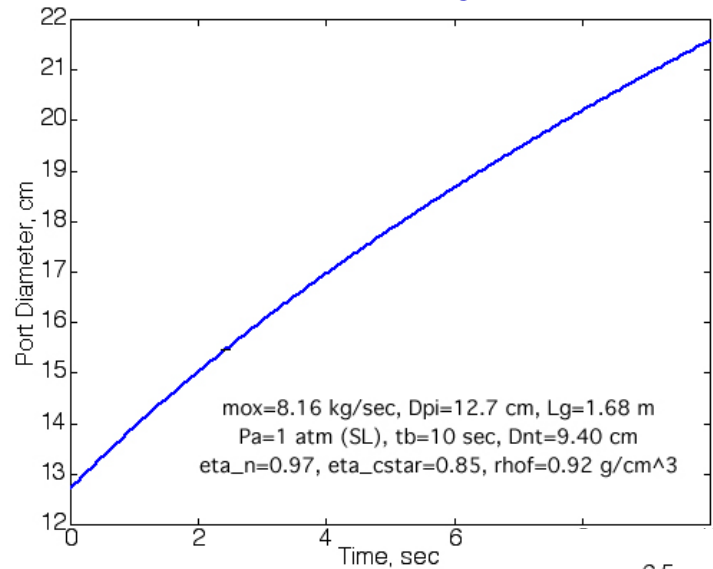
$$P(t) = \frac{(\dot{m}_{ox} + \dot{m}_f) c_{theo}^* \eta_c}{A_{nt} C_d} \quad T(t) = (\dot{m}_{ox} + \dot{m}_f) c_{theo}^* \eta_c C_{Ftheo} \eta_n$$

- Numerical integration is required for complex oxidizer flow rate schedules



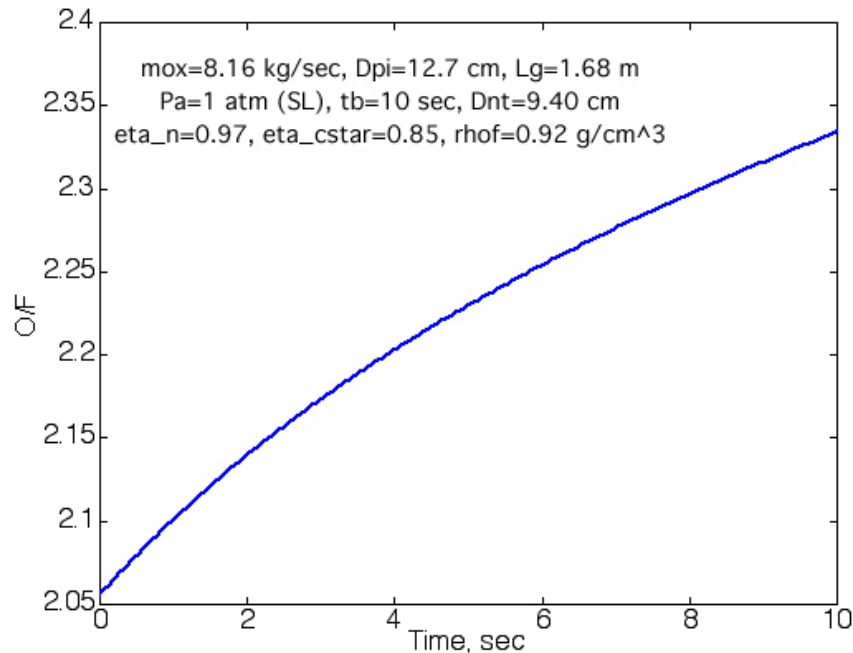
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Axial Flow Hybrid Rocket –Design Example



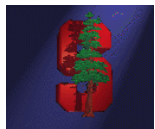
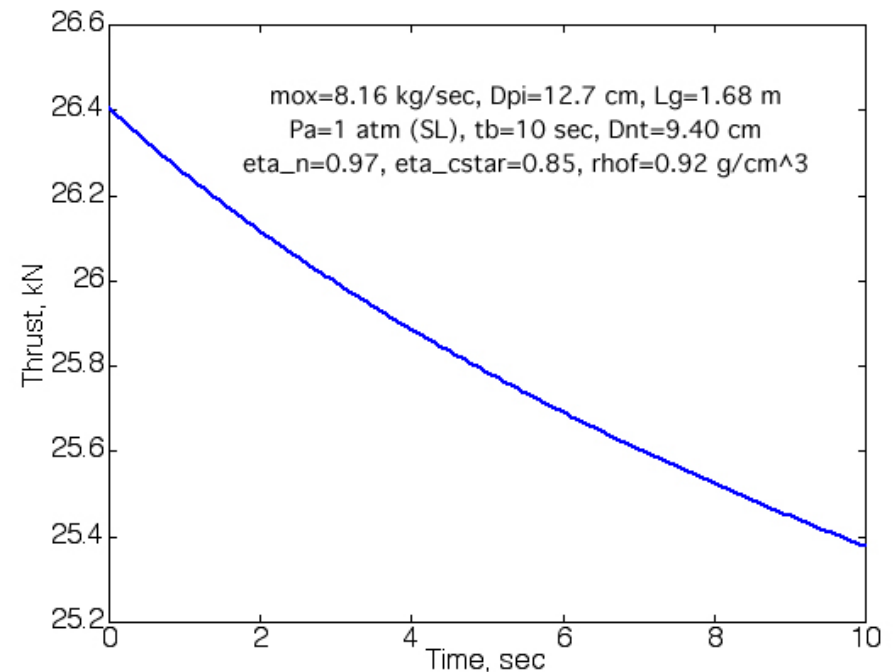
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Axial Flow Hybrid Rocket –Design Example



- *O/F* Shift: *O/F* in a hybrid rocket changes in time
 - Shift due to port opening
 - Shift due to throttling

- Thrust change is due to
 - Change in the fuel mass flow rate
 - *O/F* shift
 - Chamber pressure drop



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Axial Flow Hybrid Rocket – Scaling Laws

- For constant oxidizer flow rate, the following formula relates the initial and final port diameters

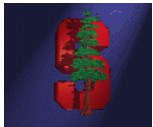
$$\left(\frac{D_{pf}}{D_{pi}} \right)^{2n+1} - 1 = \frac{(2n+1)2^{2n+1}a}{D_{pi}^{2n+1}\pi^n} \dot{m}_{ox}^n t_b$$

- The final port diameter can be solved in terms of the total oxidizer mass M_{ox}

$$D_{pf} = \left[\left(\frac{(2n+1)2^{2n+1}a}{\pi^n} \right) \frac{M_{ox}^n t_b^{1-n}}{1 - (D_{pi}/D_{pf})^{2n+1}} \right]^{\frac{1}{2n+1}}$$

- The fuel grain length can be related to the O/F

$$L = \frac{4M_{ox}}{\pi\rho_f(O/F)(D_{pf}^2 - D_{pi}^2)}$$



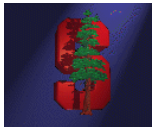
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Axial Flow Hybrid Rocket – Design Process

- Select
 - Propellants
 - Port diameter ratio
 - Structural design constraint
 - Bore stress increases with increasing diameter ratio
 - A typical value is 2
 - Note that the diameter ratio is related to the volumetric loading

$$VL = 1 - \left(\frac{D_{pi}}{D_{pf}} \right)^2$$

- The diameter ratio also determines the flux ratio during the burn
- O/F (from optimal Isp)
- Burn time
 - from optimal trajectory and constraints
 - Minimize the gravity loss under the acceleration and Qmax constraints
- Propellant mass (from mission requirement)
- Chamber pressure
- Nozzle area ratio
- Use the design equations to determine the geometrical parameters: grain dimensions, nozzle throat and exit areas



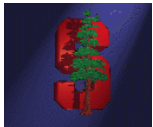
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Axial Flow Hybrid Rocket – Design Process

- The grain geometry is critical in achieving an efficient packing in a hybrid rocket system
- For liquid systems packing is relatively easy since the tank geometries can be selected freely
- For solids packing is less of an issue
 - Solids are denser
 - A wide range of fuel grain geometries are possible (fuel generation rate is proportional to the fuel surface area)
- For most applications small L/D values are desirable. Maximize the grain diameter (Written in terms of the total impulse (I_{tot}), volumetric loading (VL) and burn time (t_b))

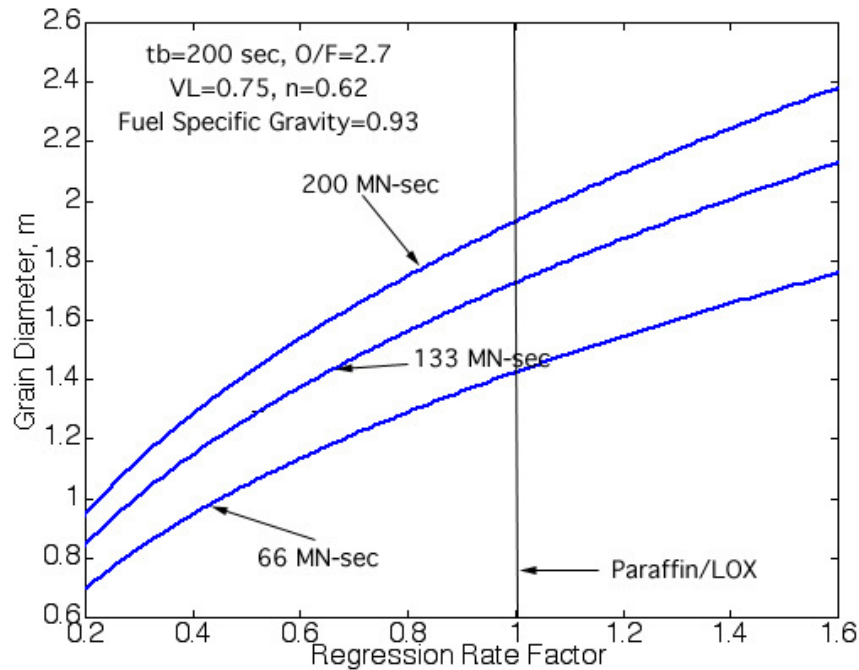
$$D_{pf} = \left[\frac{\left(\frac{(2n+1) 2^{2n+1} a}{I_{sp}^n g_o^n \pi^n} \right) \left(\frac{O/F}{1+O/F} \right)^n I_{tot}^n t_b^{1-n}}{1 - (1-VL)^{(2n+1)/2}} \right]^{\frac{1}{2n+1}}$$

- Note that grain length is $L = \frac{4}{\pi \rho_f VL} \frac{I_{tot}}{I_{sp} g_o} \frac{1}{(1+O/F) D_{pf}^2}$



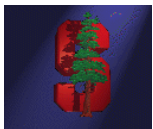
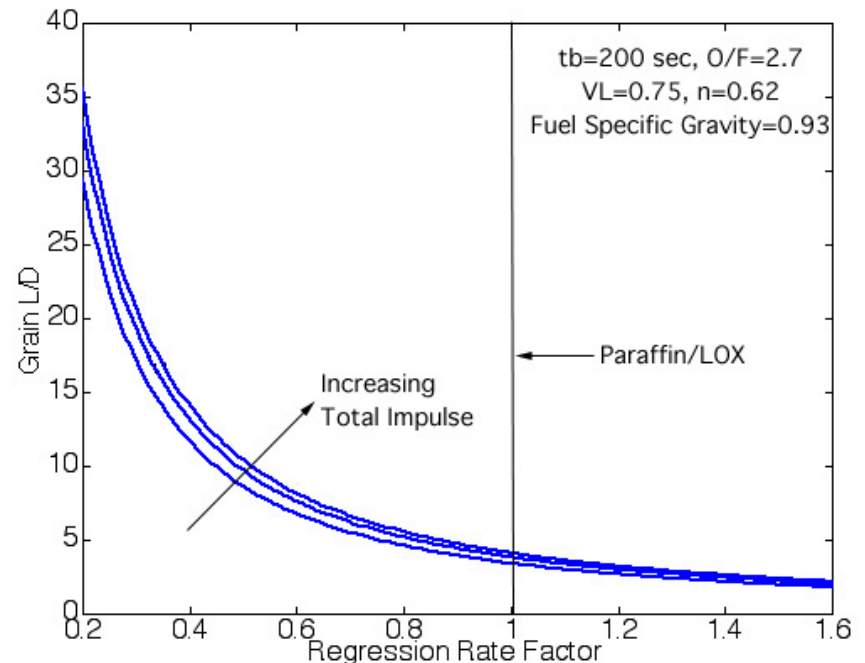
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Axial Flow Hybrid Rocket – Scaling Trends



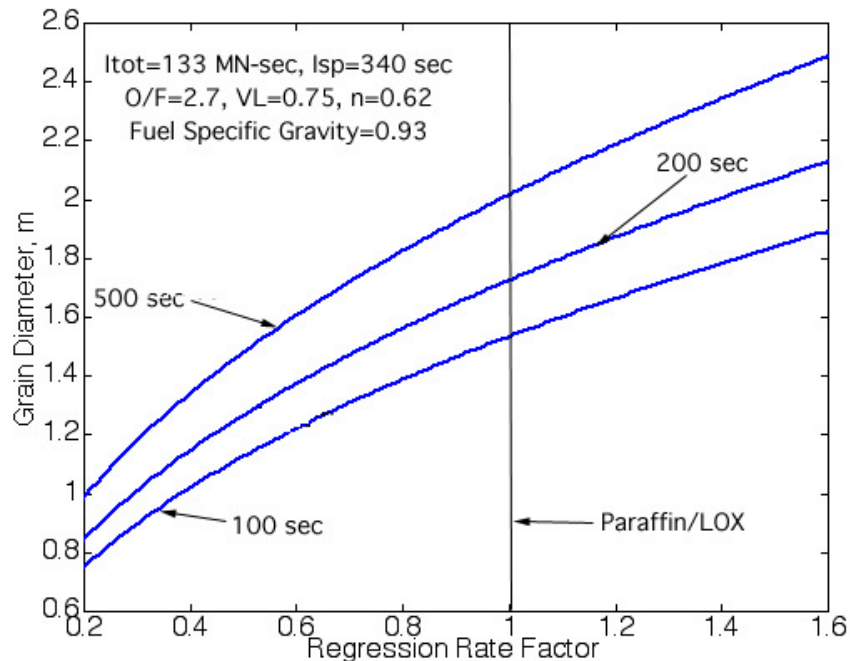
- Grain Diameter increases with increasing impulse
- Grain L/D slightly increases with increasing total impulse

- Grain Diameter increases with increasing regression rate coefficient
- Grain L/D decreases with increasing regression rate coefficient



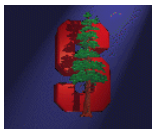
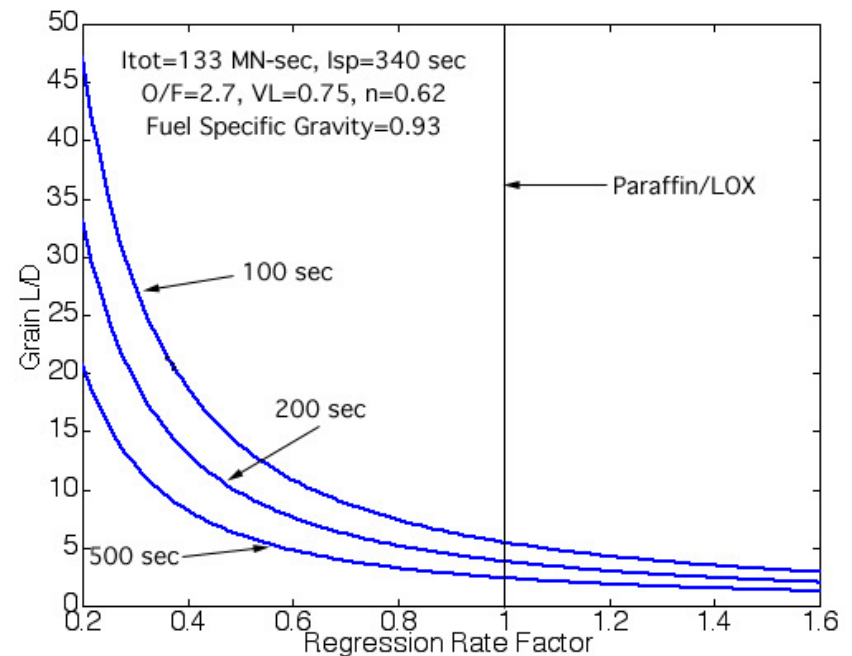
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Axial Flow Hybrid Rocket – Scaling Trends



- For slow burning fuels, L/D for a single port system is unacceptably large
- This is the driving force for multi-port designs

- Grain Diameter increases with increasing burn time (for $n < 0.5$)
- Grain L/D decreases with increasing burn time



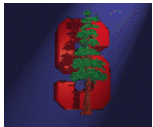
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O/F Shift

- The oxidizer to fuel ratio of a liquid rocket is directly controlled by adjusting the oxidizer and fuel mass flow rates
- The O/F of a solid system is constant (or passively programmed) since fuel and oxidizer are premixed in the solid phase
- The O/F of a hybrid rocket shifts during the operation since the fuel generation is determined by the physics and chemistry of the combustion process
- The O/F for a single circular hybrid can be written as

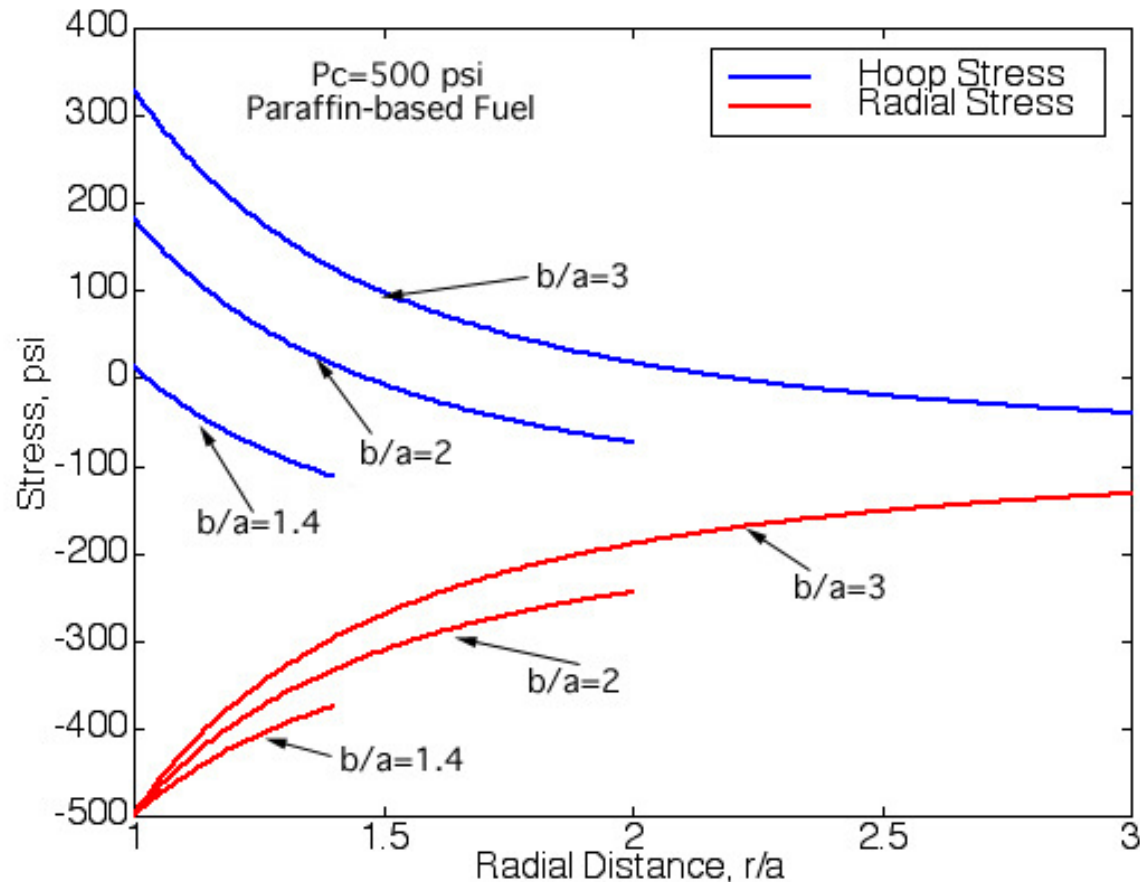
$$\frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_f} = \frac{\pi^n D_p^{2n} \dot{m}_{ox}}{\rho_f \pi D_p L a 4^n \dot{m}_{ox}^n} = \frac{D_p^{2n-1} \dot{m}_{ox}^{1-n}}{\rho_f \pi^{1-n} a 4^n \rho_f L}$$

- For typical hybrids the port exponent $2n-1$ and the oxidizer mass flow rate exponent $1-n$ are both positive ($0.5 < n < 0.8$)
- Thus O/F increases with time (diameter effect) for constant oxidizer mass flow rate. For $D_{pf}/D_{pi}=2$ and $n=0.62$, O/F shifts by a factor of 1.18. Effect of this kind of a shift on the c^* efficiency is small due to the flat nature of the c^* curve around the optimum O/F
- Also O/F increases with increasing oxidizer mass flow rate (for $n=0.62$, a throttling ratio of 10:1 changes the O/F by a factor of 2.4)
- If $n=0.5$ no shift due to port opening, if $n=1$ no shift due to throttling
- Oxidizer flow rate can be programmed to keep the O/F constant $\dot{m}_{ox} \propto D_p^{(2n-1)/(1-n)}$
- In a classical hybrid both O/F and thrust can not be controlled simultaneously
- Aft oxidizer injection allows one to schedule the required thrust profile at a constant O/F

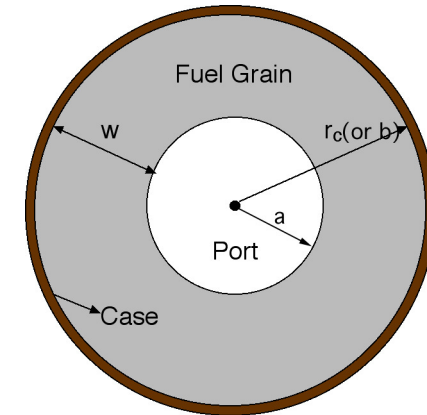


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Fuel Grain Stress Distribution-Pressure Loading

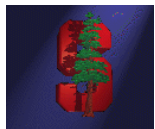


- Stress Distribution
- Pressure Loading
- Single Circular Port
- Paraffin-based Fuel



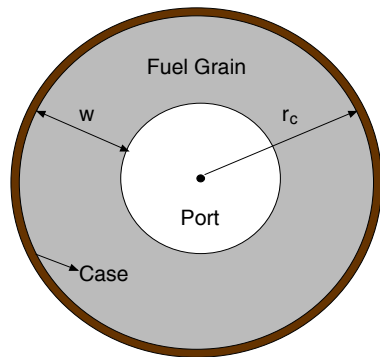
- Material approaches failure boundary as b/a increases

Failure Boundaries: $\tau = \frac{|\sigma_r - \sigma_\theta|}{2} < \tau_y$ $\sigma = \max(\sigma_r, \sigma_\theta) < \sigma_f$

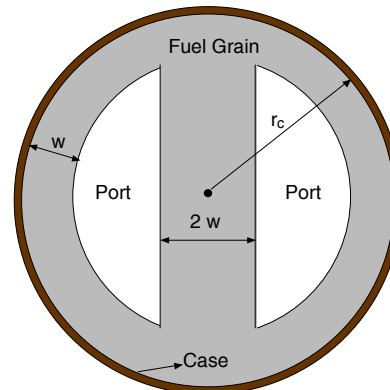


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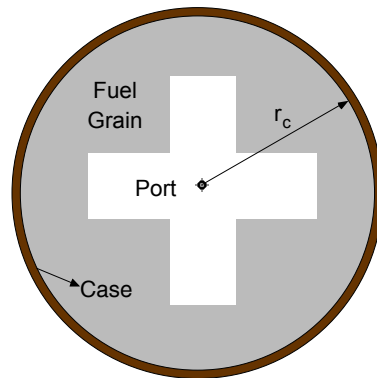
Hybrid Rocket Fuel Port Designs



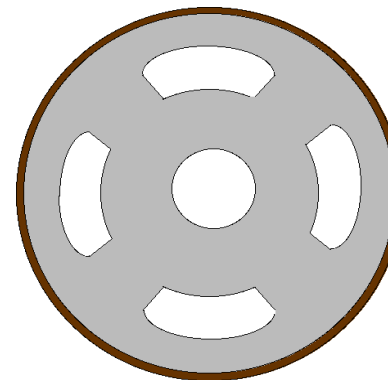
Single Circular Port



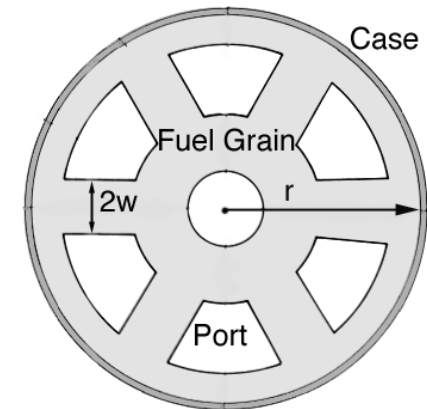
Double-D



Cruciform Port

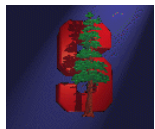


4+1 Port Design



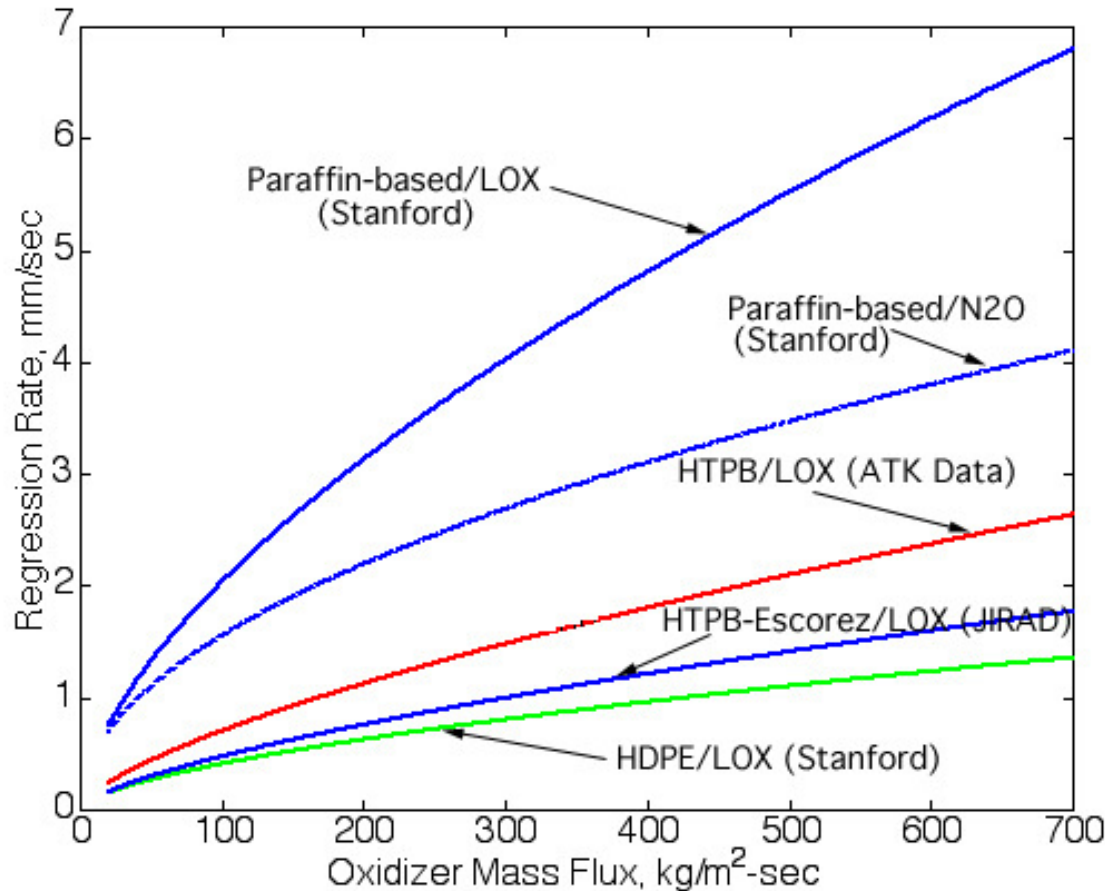
6+1 Port Wagon Wheel

Decreasing Regression Rate



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Regression Rate Data for Various Hybrid Propellants



- **HTPB/LOX:**

$$\dot{r} = 3.043 \cdot 10^{-2} G_{ox}^{0.681}$$

- **HTPB/Escorez/LOX**

$$\dot{r} = 2.061 \cdot 10^{-2} G_{ox}^{0.68}$$

- **HDPE/LOX**

$$\dot{r} = 2.340 \cdot 10^{-2} G_{ox}^{0.62}$$

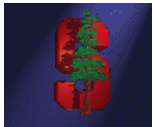
- **Paraffin/LOX**

$$\dot{r} = 11.70 \cdot 10^{-2} G_{ox}^{0.62}$$

- **Paraffin/N₂O**

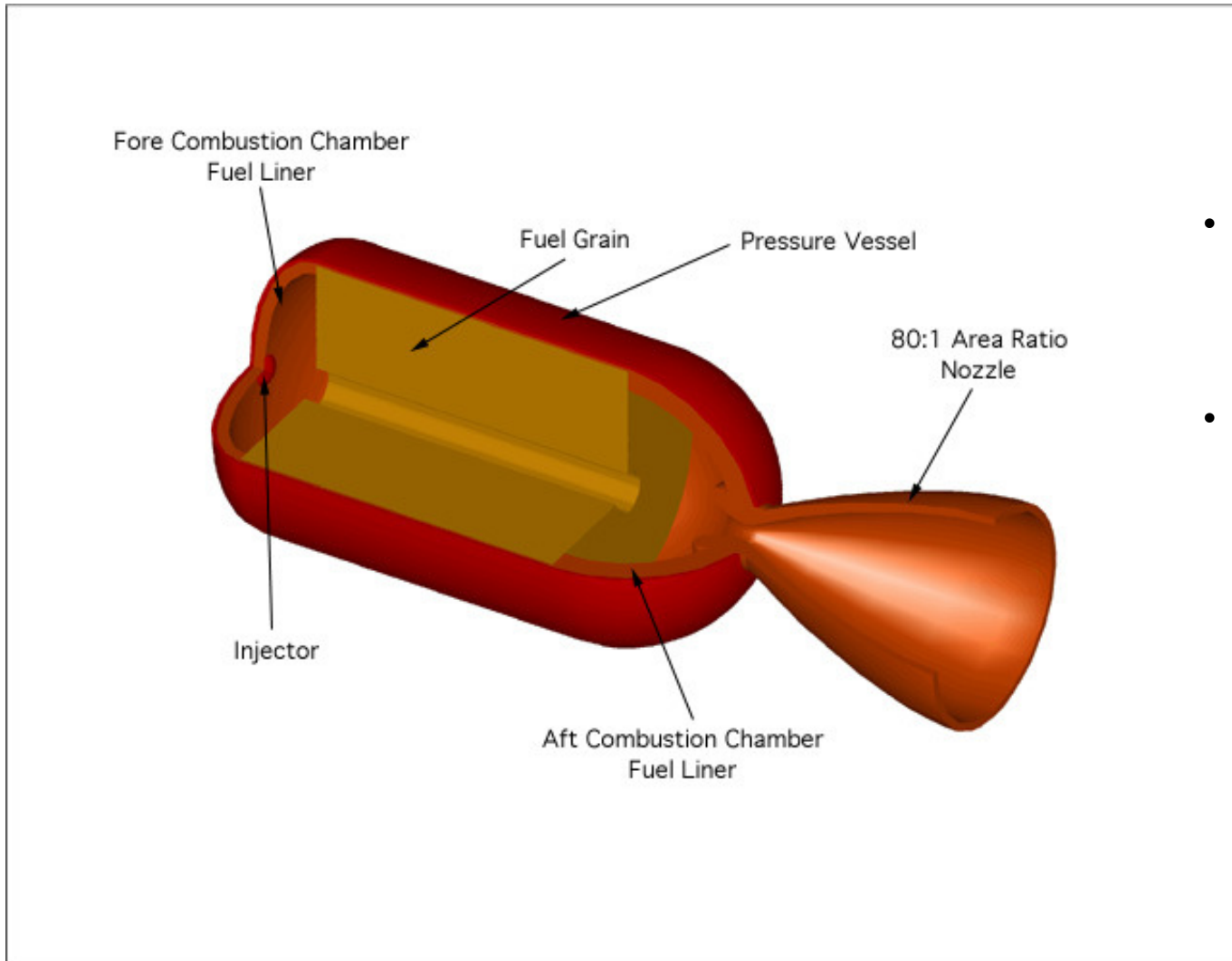
$$\dot{r} = 15.50 \cdot 10^{-2} G_{ox}^{0.50}$$

(Units are mm/sec and kg/m²-sec)



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Hybrid Rocket Internal Ballistic Design Example



- **Pre-combustion chamber: vaporization of the oxidizer (2:1 ellipse)**
- **Post-combustion chamber: mixing and reaction of the unburned fuel and oxidizer. Increase effective L^* (hemisphere)**

